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ROTATIONAL BEHAVIOR OF THE STARS AND ITS PLAUSIBLE CONSEQUENCES CONCERNING FORMATION OF PLANETARY SYSTEM II

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Su-Shu Huang
Dearborn Observatory, Northwestern University, Evanston, Illinois

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Abstract

Following the view taken in a previous paper that the statistical behavior of the observed rotational velocities of stars depends upon the braking mechanism, we have studied several statistical models based on the truncated Maxwellian distributions of original stellar angular momenta as well as on different braking processes. Observational results appear to indicate that the rate of decrease of angular momentum of a star at any moment must be proportional to the first or/and higher powers of the angular momentum itself. No constant term should occur in the braking equation.

Arguments have been advanced to show that the braking of stellar rotation leads to formation of planetary systems. Therefore the observed statistical property of early-type main-sequence stars that rotate rapidly may be used as giving an estimate for the frequency distribution of planetary systems around the late-type main-sequence stars that do not rotate significantly.

I. DISTRIBUTION OF ROTATIONAL VELOCITIES OF COEVAL STARS

Consider a rotating star that is being braked by electromagnetic force according to the mechanism of mass dissipation along the magnetic lines of force proposed by Schatzman (1962). The actual rate of decrease of the stellar spinning angular momentum depends, of course, upon both the amount of mass ejected per unit time as well as the distance from the star where ejected charged particles are decoupled from the stellar magnetic field. If the spinning angular momentum of the star expressed in terms of a dimensionless variable is x, the rate of its decrease may be written as

$$\frac{dx}{dt} = -\alpha_0 - \alpha_1 x - \alpha_2 x^2 - \cdots \tag{1}$$

where α_n's are parameters independent of x. We shall consider three simplified cases of the braking processes: All α_n's in equation (1) vanish except (1) $\alpha_0 \neq 0$, (2) $\alpha_1 \neq 0$, and (3) $\alpha_2 \neq 0$. Hence the relation between the initial value of x at t = 0 (denoted by x_0) and the value at time t are given by

$$x = x_0 - x_1 \qquad \text{for case (1),} \tag{2}$$

$$x = \Lambda x_0 \qquad \text{for case (2),} \tag{3}$$

and

$$\frac{1}{x} = \frac{1}{x_0} + \frac{1}{x_2} \qquad \text{for case (3)} \tag{4}$$

where

$$x_1 = \alpha_0 t \;, \quad A_0 = e^{-\alpha_1 t} \quad \text{and} \quad x_2 = \frac{1}{\alpha_2 t}$$

measure the strengths of braking in the three cases respectively. While x_1 increases with the braking strength, A_0 and x_2 decrease with it.

For a group of coeval stars such as found in a cluster, t is the same. Hence in the present section we consider cases in which A, x_1 and x_2 are taken as constant. Since the braking mechanism cannot reverse the direction of rotation, x is non-negative. Thus in case (1) x must be equal to zero if $x_1 > x_0$. On the other hand if x_0 is non-zero, x does not vanish in cases (2) and (3), as long as the braking force remains finite.

For a group of stars such as those found in one spectral class, the mass, radius, and radius of gyration may be approximately taken as equal for all members. If so, the angular momentum is directly proportional to the equatorial rotational velocity. Since the angular momentum x is expressed in a dimensionless variable, we can use the same to denote the equatorial rotational velocity. In other words, expressed in the unit of angular momentum x denotes the angular momentum, and expressed in the unit of equatorial rotational velocity x denotes the equatorial rotational velocity. The unit of angular momentum and the unit of equatorial rotational velocity are of course connected by a simple relation. Actually there is the evidence coming from different fields of investigation that the magnetic activity occurs at the pre-main-sequence stage in which the star contracts. Hence, the radius can not be said to be constant in the period of momentum dissipation. However, for coeval stars in the same mass range the linear relation between equatorial rotational velocity and angular momentum is maintained at all times, although the linear factor does change with time as the stars evolve.

We may assume as in Paper I (Huang 1965) that x_o is distributed according to a truncated Maxwellian distribution, namely,

$$f(x_o)\,dx_o = \frac{4\gamma}{\sqrt{\pi}}\, x_o^2 e^{-x_o^2}\, dx_o\,, \qquad 0 \leq x_o \leq x_c \qquad (5)$$

where the numerical factor γ is introduced to normalize the distribution function. The justification of this assumption has been briefly indicated in Paper I. But it will be more thoroughly examined in a forthcoming paper. If we should take stellar objects of high angular momenta from rotating stars to binaries as forming a single group, the rotating stars will have small angular momenta as compared with the rest of the group. Hence the point of truncation, x_c, will be a small value. On the other hand, if rotating stars and binaries form two distinct groups which are not formed by the same processes, the distribution of angular momenta of rotating stars and that of binaries will be independent of each other. In such a case x_c will be large. In the present investigation we shall leave x_c (and consequently γ) unspecified.

Our problem is first to find the distribution of x after all stars in the group have been braked according to equation (1) for a definite length of time, and next to obtain the distribution of the observed rotational velocity, y, given by

$$y = x \sin i, \tag{6}$$

where i denotes the inclination of the equatorial plane. This can be done because the distribution function of x, say F(x), and that of y, say $\Phi(y)$, are related by the following integral equation (Kuiper 1935)

$$\Phi(y) = y \int_y^\infty \frac{F(x)\,dx}{x(x^2-y^2)^{1/2}}, \tag{7}$$

if the equatorial planes are oriented in space at random. Since y is an observable quantity, the distribution of y thus computed can be compared with the observed distribution. The comparison determines the unit of y (and consequently x) that is unspecified in the present calculation.

In Paper I we have studied case (1). The result shows that the distribution of x and consequently that of y assume finite values respectively at x = 0 and y = 0 as soon as the stars have been braked (i.e., $x_1 > 0$). As x_1 increases, the number of stars with y = 0 increases rapidly. For a large enough value of x_1, practically all stars will have $x = 0$, and therefore $y = 0$.

We shall now study cases (2) and (3). It follows from equation (3) that x in case (2) is distributed, just like x_0, according to the Maxwellian distribution because there is only a change in the scale of the independent variable. It can be easily seen that the corresponding distribution of y is also changed only by a scale factor. Since the scale factor is left free in the present calculation, we can omit this scale factor and write down the distributions of y for case (2) as

$$\Phi_1(y)\,dy = \frac{4\gamma}{\sqrt{\pi}}\, y^2 dy \int_0^{\theta_1} e^{-(y\sec\theta)^2} \sec^2\theta\, d\theta, \qquad 0 \leq y \leq y_1 \tag{8}$$

where

$$\theta_1 = \cos^{-1}\frac{y}{y_1}$$

and y_1 is the upper limit of y. The integral can be numerically evaluated and the case with $y_1 \to \infty$ is shown in Figure 1 by the curve marked by the numeral 1. Since in the present paper we shall consider several distribution functions of y under different statistical assumptions, the function $\Phi(y)$ is labeled by a subscript in order to avoid confusion. $\Phi(y)$ without any subscript is reserved for the observed function and for general usage. The characteristic property of the function $\Phi_1(y)$ is that it vanishes as $y \to 0$ no matter how strong is the braking mechanism. Here we see that the result derived in case (2) is greatly different from that derived in case (1) in the range of small y.

Finally we may inquire whether the behavior of the distribution at y = 0 will be different if the rate of change of spin angular momentum decreases more than linearly with x. This leads us to case (3) of the braking process. Equations (4) (5) and (7) yield the following results:

$$\Phi_2(y)\,dy = \frac{4\gamma}{\sqrt{\pi}}\, x_2^4\, y^2 \int_0^{\theta_2} \frac{\cos^2\theta}{(x_2\cos\theta - y)^4}\, e^{-\left(\frac{x_2 y}{x_2\cos\theta - y}\right)^2} d\theta\,, \qquad (9)$$

where

$$\theta_2 = \cos^{-1}\frac{y}{y_2} \qquad \text{and} \qquad \frac{1}{y_2} = \frac{1}{x_2} + \frac{1}{x_c}\,.$$

It can then be seen that the distribution $\Phi_2(y)$ vanishes also at y = 0. Figure 2 illustrates the behavior of $\Phi_2(y)$ for a few cases of y_2 (0.8, 1.0, 1.5, 3.0, as labeled) with $x_c \to \infty$. Since y_2 decreases as the braking strength increases, the curve that corresponds to the largest y_2 denotes the case of the least braking. The distributions show markedly different forms for different degrees of braking. Note the sharp drop of the distribution curves at the upper end of y such as shown in the cases corresponding to y_2 = 0.8 and 1.0.

In general we can show in a similar manner that if $dx/dt = - \alpha_n x^n$ the distribution of y always vanishes at $y = 0$ as long as $n > 1$. On the other hand if $0 \leq n \leq 1$, it assumes a finite value at $y = 0$ because rotation of some stars is completely stopped by braking.

II. OTHER FACTORS THAT AFFECT THE DISTRIBUTION FUNCTION

So far we have considered only the statistical behavior of rotational velocities for stars of the same mass and of the same age. While this may be roughly true for stars of a given spectral type in a cluster, it is not so for the field stars even of the same spectral and luminosity classification, as they may be formed at different periods. Consequently the observed distribution of y is further influenced by this time factor. Another complication that may modify the distribution of y is the non-constancy of α_n's in different stars. This quantity depends on many factors, such as the average field strength of magnetic activity on the surface of stars, the magnetic field in interstellar space, and the rate of mass ejection. In what follows we shall consider only case (2) of the braking process but do not insist that α_1 should be a constant for all stars.

Let us first examine the time factor. We may divide the problem into two idealized cases: (1) The braking mechanism lasts a long time comparable to the time scale of nuclear evolution but the rate of angular momentum dissipation is constant for all stars. This is equivalent to saying that t in the equation A = exp. $(-\alpha_1 t)$ may be regarded as being distributed according to

$$\xi(t)\,dt = \frac{dt}{T}\,, \qquad 0 \leq t \leq T. \tag{10}$$

The constant, T, is the total time that the braking mechanism is operative. According to this simple model those rotating stars that we observe are still being braked right now. (2) The braking mechanism takes place in a short time interval, say of the order of the time scale of the pre-main-sequence gravitational contraction, but α_1 varies. In this model, the braking mechanism in stars we study has already stopped. This is equivalent to saying that the ages of stars we now study are all much greater than T, although T may not be constant for all stars. In such cases the effect of braking on the distributions of x and consequently of y depend upon the distribution of the integral

$\tau = \int_0^T \alpha_1(t)\,dt$ integrated over the time of braking. A reasonable assumption is that τ is distributed according to a Gaussian error function centered at a certain value, say $(\tau_0 + \tau_1)/2$ with a certain dispersion. Such a distribution may be represented approximately by a step function, namely,

$$\eta(\tau)\,d\tau = \frac{d\tau}{\tau_1 - \tau_0}, \quad \tau_0 \le \tau \le \tau_1 \tag{11}$$

and vanishes elsewhere.

If we now assume that x and x_0 are related by equation (3), the two cases lead to the same mathematical problem, because in the first case we are to find the distribution of x when t is distributed according to equation (10) and x_0, t, and x are related by $x = x_0 \exp.(-\alpha_1 t)$, and in the second case we are to find the distribution of x when τ is distributed according to equation (11) and x_0, τ, and x are related by $x = x_0 \exp.(-\tau)$. ← In both cases x_0 is distributed according to equation (5). Mathematically the second case includes the first case, (with $\tau_0 = 0$), since α_1 in the first case can always be set equal to 1 by suitably choosing the unit of time. For this reason we only have to compute the distribution function F(x) of x in the second case.

The distribution function F(x) of angular momenta (or of equatorial rotational velocities) can be obtained by a standard method (Chandrasekhar and Münch 1950). We have

$$F(x)\,dx = \iint f(x_0)\,\eta(\tau)\,dx_0\,d\tau \tag{12}$$

where the integration domain on the (x_0, τ) plane is over the strip

$$x \leq x_0 e^{-\tau} < x + dx, \tag{13}$$

bounded by $x_0 = 0$, $x_0 = x_c$, $\tau = \tau_0$, and $\tau = \tau_1$. Equation (12) can be integrated to give

$$F(x)dx = \frac{\gamma}{\ln\lambda_1 - \ln\lambda_0} \frac{dx}{x} \left[I(\lambda_1 x) - I(\lambda_0 x) \right], \quad 0 \leq x \leq x_c/\lambda_1$$

$$= \frac{\gamma}{\ln\lambda_1 - \ln\lambda_0} \frac{dx}{x} \left[I(x_c) - I(\lambda_0 x) \right], \quad x_c/\lambda_1 \leq x \leq \tag{14}$$

if we let

$$\lambda_0 = e^{\tau_0} \quad and \quad \lambda_1 = e^{\tau_1} \tag{15}$$

and define the function $I(\omega)$ as follows

$$I(\omega) = \frac{4}{\sqrt{\pi}} \int_0^{\omega} t^2 e^{-t^2} dt \tag{16}$$

The distribution function $\Phi(y)$ of the observable quantity y can be obtained by substituting equation (14) into equation (7). Simple calculations bring the resulting equation into the following form

$$\Phi_3(y)dy = \frac{1}{\ln\lambda_1 - \ln\lambda_0} \frac{dy}{y} \Big\{ \gamma\left[J(\lambda_1 y, \xi_1) - J(\lambda_0 y, \xi_1) \right]$$

$$- \gamma\left[J(\lambda_0 y, \xi_2) - J(\lambda_0 y, \xi_1) \right] + (\sin\xi_2 - \sin\xi_1) \Big\},$$

$$0 \leq y \leq x_c/\lambda_1 \tag{17}$$

$$= \frac{1}{\ln\lambda_1 - \ln\lambda_0} \frac{dy}{y} \left[\sin\xi_2 - \gamma\, J(\lambda_0 y, \xi_2) \right],$$

$$x_c/\lambda_1 \le y \le x_c/\lambda_0 ,$$

where

$$\xi_1 = \cos^{-1}\frac{\lambda_1 y}{x_c} \quad \text{and} \quad \xi_2 = \cos^{-1}\frac{\lambda_0 y}{x_c} .$$

and the function $J(\omega, \varphi)$ is defined by

$$J(\omega, \varphi) = \int_0^{\varphi} I(\omega \sec\theta) \cos\theta \, d\theta, \qquad (18)$$

while the function I(x) in equation (18) is given by equation (16). It can easily be seen that whatever the values of λ_0 and λ_1, $\Phi_3(y)$ vanishes always at y = 0.

Superficially the function $\Phi_3(y)$ involves two parameters λ_0 and λ_1. Actually only one is significant because we can always change the scale of y to make $\lambda_0 = 1$. Figure 1 illustrates a few cases of $\Phi_3(y)$ for $\lambda_0 = 1$ and $\lambda_1 = 1.5$, 2, 3, 4, and 5 with $x_c \rightarrow \infty$ (consequently $\gamma = 1$). The value of λ_1 of each curve is marked in the figure. The case $\lambda_0 = \lambda_1 = 1$ denotes simply the function $\Phi_1(y)$ as given by equation (8). Because y in the diagram and y actually observed differ by a scale factor which is not a priori known, it follows from the behavior of the curves in Figure 1 that we cannot distinguish different distributions arising from different values of λ_1, especially if we remember the approximate nature of the observed distribution function which can be obtained only in the form of a histogram. This point can be seen most clearly if we stretch the abscissa of the curve corresponding to $\lambda_1 = 5$ by a factor of two and shrink the ordinate by the same factor. The resulting curve will be very close to the curve corresponding to $\lambda_1 = 1$. Hence the factors considered in this section cannot be determined by the observed distribution of rotational velocities, at least with the accuracy of present data.

III. COMPARISON WITH OBSERVED RESULTS

Let us now turn our attention to the observational results. The early data (Huang 1953) appear to agree with case (1) of braking process. However, because of the low dispersion of plates that were used for study and because of the crude method for measuring the rotational velocities, the resolution of the observed distribution function is very low. This is especially true in the region where $y \to 0$ as has been emphasized. Since then several investigators have measured the rotational velocities with higher precision. It appears that their results show $\Phi(y) \to 0$ as $y \to 0$. Deutsch (1965 a, b) and Kraft (1966) favor the Maxwellian distribution of x. In any case the braking process appears to indicate $n > 1$.

Let us now consider these cases from the theoretical point of view. Case (1) assumes that the braking of rotation is constant independent of the rotational velocity while case (2) assumes that the braking force is proportional to the rotational velocity itself. If we now consider Schatzman's mechanism of braking, it is quite obvious that the angular momentum carried away by ejected charged particles is likely proportional to the angular velocity of the star at that moment. This would mean the situation implied in case (2), if the rate of mass ejected and the magnetic field strength are independent of rotation. Actually the magnetic activity that brakes stellar rotation results from rotation and convection, so that the rate of decrease in x should be more than proportional to x, thereby corresponding to $n > 1$.

If $\alpha_0 = 0$, the present statistical models expect no stars that are absolutely free from rotation, although the rotational velocities may be very small. In other words, the braking mechanism can not stop the rotation completely. Our own sun with a rotational velocity of 2 km/ sec at its equator is a case in point. However, because of the measurable limit no rotation of main-sequence stars of spectral types later than F5 has been detected (Struve 1930). The strong braking suffered by these late-type stars may be due to either large α_1 or long T. The latter possibility becomes apparent if we remember that a convection zone which may produce the magnetic activity exists near the surface in these stars. Indeed this fact led Schatzman (1962) to propose the magnetic braking theory of stellar rotation.

Another important factor involved in the braking process is the efficiency of angular momentum dissipation. If the charged particles ejected along the magnetic lines of force come back along the same lines of force, no angular momentum of the star will be lost. This is the case with charged particles in the Van Allen belt of the earth. An efficient way of dissipation requires the presence of a cold medium in the surroundings where particles are not ionized. Stellar angular momentum can then be efficiently transported to this medium. While a remnant medium is expected to be present in the neighborhood of a newborn star, it will not remain un-ionized near a hot star. This again discriminates the braking of early-type stars and helps explain why they are still rotating rapidly while no late-type main-sequence single stars show projected rotational velocity greater than, say 10 km/sec.

As observing and recording techniques improve, it may be expected that slow rotation of these stars will be found. But the suggestion advanced in Paper I that the difference in rotational behavior of main-sequence stars of various spectral types arises from the braking strength or/and the duration in which braking is operative remains our basic view.

It follows that the mean rotational velocity along the spectral types, or equivalently the observed stellar angular momentum distribution with respect to stellar masses (Kraft 1966), gives a measure of the dissipation mechanism of angular momenta for stars of different masses.

As pointed out recently by Kraft (1966), one of the most puzzling phenomena about stellar rotation is the fact that a few groups of stars that fall in the spectral range known for their rapid axial rotation show unusually small rotational velocities. For example it has been found that there are two sharp peaks of overpopulation of narrow-lined stars at B2V and at AOV (Conti 1965). Based on the concept of the present paper it must be the result of a strong braking strength or a long braking duration, or both. We may suggest that the braking of early-type stars is in general limited only to the period when the stars are undergoing gravitational contraction in a state of convective equilibrium (Hayashi 1961). Once the energy transported in the envelope becomes radiative, the braking mechanism stops. This

is clearly seen from Poveda's (1964) investigation on the flare stars. If, however, some groups of stars maintain their magnetic activity even after the main-sequence stage has been reached, the braking mechanism will continuously be effective for a long period of time. The result is to reduce the rotational velocities to small values. In other words, the values of A in equation (3) for these groups of stars are small. Thus, it is interesting to note that some of them do indeed show large magnetic field strengths on the surface. In such cases A is small because α_1 is large. In other cases A may become small because t is large. For example, Bidelman (1960, see also Fowler, et al 1965) has suggested that the Ap stars which show in general slow rotation (Deutsch 1965) are post-red giant stars. If so, the slow rotation results from a prolonged dissipation of the angular momentum.

IV. OCCURRENCE OF PLANETARY SYSTEMS

Our experience of the planetary system is limited because we have actually observed only one system. Therefore when we try, as in the present series of papers, to discuss occurrence of planetary systems in general, it is important to know what we mean by planetary systems. From the special case of our own system we may suggest a general definition that any planetary system should possess the following three characteristics: (1) a small mass ratio of the total mass of the entire planetary system to the mass of the central star; (2) a large angular momentum; (3) the majority of its members moving in nearly coplanar and circular orbits.

From this definition one can immediately realize that although binaries and planetary systems share the second characteristic, they diverge with respect to the third characteristic. As we know, there is a considerable fraction of binaries that exist as a part of the multiple system. While some degree of alignment may exist among the orbital planes of different pairs in a multiple system, it is far from being conspicuous (Worley 1966). In fact, among the many multiple systems so far studied, only two (44 / Bootis and 6 Trianguli) show close alignment of the orbital planes of the member stars.

This statistical property is definitely at variance with the coplanar condition of the planetary system. Also binaries differ from planetary systems by their orbital eccentricities. While the former can have any value from 0 to 1, the latter is supposed to show dominantly near-circular orbits. Thus, the unseen companion of the Barnard star inferred from the proper motion (van de Kamp 1963) may not be regarded as forming a planetary system, as has been emphasized by Kumar (1964), because the eccentricity is equal to 0.6, a fairly large value.

In the past when we discussed the possible occurrence of planetary systems in space, we usually linked it with the population of binary systems because of the angular momentum consideration (Kuiper 1935). It followed that planetary systems must be common because binaries are numerous. However, from what we have just said, such an inference can not be regarded as valid. Hence the problem of emergence of planetary systems must be examined in the light of their characteristics. What made the mass ratio so small? What made the planets lie nearly in one plane? What made them move in nearly circular orbits? All these characteristics point to our suggestion that formation of planetary systems is closely related to the braking of stellar rotation.

Since the matter that is to form a planetary system is but the remnant of star formation, the mass ratio is naturally small. We may envisage the initial stage of contraction as a gravitational collapse (Gaustad 1963). Matter falls freely toward the central star. But the in-falling matter will slow down after the star begins to shine and thereby provide a remnant medium around the star in the early stage. This medium cannot stay very long as it is, because particles in it will either fall into the star or escape into space. However, its acquisition of angular momentum from the star as a result of the braking process changes the situation. The gaseous medium collapses into a rotating disk which can maintain itself for a long time such that planets may condense from it. That planets emerge from a rotating disk of gases and dust insures circulatory motion in a single plane. Therefore, in order to find the frequency of planetary systems in space, we should investigate how often a rotating disk of gases and dust may be formed around a star. This is the basic concept that underlies the present investigation. This reasoning immediately leads to the relation between braking of stellar rotation and formation of planetary systems.

Formation of the gaseous rotating ring or disk is actually a common phenomenon in nature. Through the emission lines we have observed gaseous rings in the rapidly rotating Be stars. The mass in the ring comes presumably from the star itself as a result of rotational breakup (Struve 1931). We also observe the emission lines due to the rotating ring around the more massive component in many an Algol-type eclipsing binary (Joy 1942, 1947, also Sahade 1960). The matter of the ring is believed to have been ejected by the less massive component whose size approaches the limiting value compatible with the binary system. Of course the rotating disk from which planets are to emerge must be considerably denser than what is found in the rings of the Be stars and in Algol-type binaries. However, there is the evidence that dense disks do exist in some binaries (Huang 1963, 1966). All these phenomena indicate that circulatory motion will result as soon as there is an adequate supply of angular momentum and provide a strong justification for formation of a rotating disk around the star after the latter has been braked. If the disk formation and the consequent emergence of planets are a natural sequence of events following the braking process, we are for the first time able to make a quantitative estimate of the frequency of occurrence as well as a rough idea of the nature of planetary systems from the statistical behavior of stellar rotation.

For definiteness we focus our attention on planetary systems around the G type main-sequence stars. Assume the angular momenta per unit mass, h, of all G stars before braking to be distributed in the same way as those of the early-type main-sequence stars. Since the effect of braking on the early-type stars is small, we may take what is observed of them now as representing the original distribution of h. On the other hand, practically all angular momenta of G stars have been transported out by the braking mechanism. If we now assume that all the angular momentum is absorbed in the planetary system, we obtain the distribution function $\psi(\Omega)d\Omega$ of Ω, the angular momenta of planetary systems around main-sequence G stars. Interestingly, the angular momentum of our own planetary system falls not too far away from the maximum point of $\psi(\Omega)$. Of course, in looking at this agreement we must bear in mind two factors: First, the original distribution of angular momentum before braking likely depends upon the mass of the star. This is especially true if the stellar angular momentum results from random motion of the pre-stellar medium. Secondly, a part of dissipated angular momentum may be lost to space together with escaping matter. However, none of these considerations will likely change the order of magnitude of our estimate.

In order to give a simple description of a planetary system we may introduce a concept of the "equivalent planet" as a fictitious planet that moves in a circular orbit of radius, a, and that possesses the total mass, m, and the total angular momentum, Ω, of the entire planetary system, the parent star being excluded as a member of the system. Hence we have

$$\Omega = m(GMa)^{\frac{1}{2}} \tag{19}$$

where M denotes the mass of the parent star and is known. Knowing $\psi(\Omega)$, we obtain the distribution of $ma^{\frac{1}{2}}$. Now if log m is plotted against log a, the diagram may be divided by a series of straight lines

$$\log m + \tfrac{1}{2} \log a = C \tag{20}$$

where C is a parameter labeling these lines. Therefore the probability of finding the equivalent planet in the different strips between the lines given by equation (20) may be computed. In this way we are able to learn something about the probability distribution with respect to the mass and size of planetary system of any spectral type of the main-sequence stars. Because the angular momentum of our own planetary system is not far from the maximum point of $\psi(\Omega)$, the position of its equivalent planet which lies somewhere between Jupiter and Saturn falls in the expected region in the log m - log a diagram.

While we admit that binaries and planetary systems belong to two different categories of stellar objects, there remains the question of why the evolution of a gaseous condensation is not unique but may follow one or the other path so that it will end up either as a binary (or multiple) system or a planetary system. The difference likely arises from many factors but the dominant ones perhaps are (1) the angular momentum per unit mass, h, and (2) the total mass of the condensation. If h is very large, the condensation just cannot contract to form a single rotating body. Two or more pre-stellar nuclei may first be formed that lead eventually to two or more stars (Huang 1957). It becomes a binary or multiple system if the total dynamical energy turns out to be negative. Since the stellar matter has never undergone the stage of being a rotating disk, the binary or multiple system does not necessarily show the circular and coplanar characteristic. Accordingly, a non-rotating star (1) may be intrinsically single, (2) may have an unseen binary companion, or (3) may possess a planetary system. The nature of the unseen companion has been recently discussed by Kumar (1966).

Finally it may be noted that while the present analysis is based for definiteness on the braking mechanism suggested by Schatzman, the same result will be obtained if the angular momentum transport follows what has been described by Hoyle (1960) in connection with his theory on the origin of the solar nebula, because we are concerned here only with the consequence of braking but not with the manner of braking.

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